

Verifying Transmission Phase Measurements at Millimeter Wavelengths Using Beadless Air Lines

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Abstract—This paper presents a simple method for verifying the phase of transmission measurements – for example, as made using instruments such as Vector Network Analyzers (VNAs). Examples are given that show the method being used at millimeter wavelengths (i.e. at frequencies up to 65 GHz) in the 1.85 mm coaxial line size. These examples use measurements made independently at the national measurement institutes of Japan and the United Kingdom. An analysis is also given showing the overall uncertainty for the method. It is hoped that the method, along with the uncertainty analysis, will be suitable for use by other laboratories seeking verified and/or accredited measurements of this type.

Index terms—vector network analyzers, phase measurement, verification, lossy coaxial lines.

I. INTRODUCTION

Precision coaxial unsupported (beadless) air lines are used routinely as primary reference standards of impedance at RF and microwave frequencies [1-3]. In this paper, we use these lines as phase standards against which the phase of transmission measurements can be checked and verified. The air lines are first characterised in terms of their measured mechanical dimensions and measured electrical loss (i.e. attenuation). The electrical length of the line is then computed from the phase of the transmission measurements made by the system being verified (e.g. a VNA) and this is then corrected to take into account the measured electrical loss. The corrected electrical length is then compared with the measured mechanical length of the line. If the electrically determined length agrees with the mechanically determined length (to within the stated uncertainties of the measurements), then this verifies the phase measurements.

The above method has previously been proposed elsewhere (see, for example [4, 5]). However, in this paper, we extend this work by:

- i) Applying the method at millimetre-wave frequencies (to 65 GHz in the 1.85 mm coaxial line size);
- ii) Simplifying the calculation of electrical length (via the experimentally determined phase constant for the line);
- iii) Establishing an uncertainty budget for the method that can be adopted by other laboratories using this method;
- iv) Testing the reproducibility of the method by using measurements made independently by two national measurement institutes – NMIJ, Japan, and NPL, UK.

II. PROCEDURE

The details of the procedure are as follows:

1. Choose a beadless air line of the correct line size and fitted with the correct connectors.
2. Verify the characteristic impedance, Z_0 , of the air line using dimensional measurements of the radii of the center conductor, a , and outer conductor, b , using:¹

$$Z_0 \approx 59.939\ 045 \times \ln\left(\frac{b}{a}\right) \quad (1)$$

These measurements can be made using Air Gauging [6] and/or Laser Gauging [7] techniques.

3. Determine the mechanical length of the line, l_m , using dimensional measurements with

¹ This checks that the characteristic impedance of the line is sufficiently close to the system impedance (e.g. 50 ohms).

traceability back to the SI base unit of length, the metre.

4. Determine the complex-valued S -parameters of the line, at a range of frequencies, using the measurement system that is to be verified (e.g. a VNA).
5. Check that the measured reflection coefficient magnitudes, $|S_{11}|$ and $|S_{22}|$, are sufficiently small to approximate the match condition for the line at all measured frequencies.²
6. Calculate the average magnitude voltage transmission coefficient, $|VTC|$, at each frequency, using:

$$|VTC| = \frac{|S_{21}| + |S_{12}|}{2} \quad (2)$$

7. Use the calculated $|VTC|$ values to determine the attenuation constant, α , of the line using:

$$\alpha = \frac{-\ln|VTC|}{l_m} \quad (3)$$

8. Use the calculated values of attenuation constant to determine the resistivity, ρ , of the line at each frequency, f , using [8]:

$$\rho \approx \left[\frac{200\alpha b}{1+(b/a)} \right]^2 \frac{\pi}{\mu_0 f} \quad (4)$$

where μ_0 is the permeability of free space ($= 4\pi \times 10^{-7}$ H/m).

9. Use the determined resistivity to calculate a value for the inductance per unit length, L , of the line (see, for example, [9-11]):

$$L = L_0 + \frac{R}{\omega} \quad (5)$$

where

$$L_0 = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \quad (6)$$

and

$$R = 2\omega L_0 d_0 \left(1 - \frac{k^2 a^2 F_0}{2} \right) \quad (7)$$

where k is the angular wavenumber ($= \omega/c$ rad/m, where $c = 299\,792\,458$ m/s and ω is the angular frequency, $2\pi f$).

$$F_0 = \frac{\left(\frac{b}{a}\right)^2 - 1}{2 \ln\left(\frac{b}{a}\right)} - \frac{\left(\frac{b}{a}\right) \ln\left(\frac{b}{a}\right)}{\left(\frac{b}{a}\right) + 1} - \frac{1}{2} \left(\frac{b}{a} + 1 \right) \quad (8)$$

$$d_0 = \frac{\delta_s \left(1 + \left(\frac{b}{a}\right) \right)}{4b \ln\left(\frac{b}{a}\right)} \quad (9)$$

where

$$\delta_s = \sqrt{\frac{2\rho}{\omega\mu_0\mu_r}} \quad (10)$$

with $\mu_r \approx 1$, for non-magnetic lines.

10. Use the calculated value of L to determine the phase constant, β , using [8]:

$$\beta \approx 2\pi f \sqrt{LC_0} \quad (11)$$

where C_0 is the capacitance per unit length of a lossless line³, and is given by:

$$C_0 = \frac{2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{b}{a}\right)} \quad (12)$$

11. Calculate the average voltage transmission coefficient phase, Θ_{VTC} , at each frequency, from the measured phase of S_{21} and S_{12} , using:

$$\Theta_{VTC} = \frac{\Theta_{S_{21}} + \Theta_{S_{12}}}{2} \quad (13)$$

² This ensures that the connections at the ends of the line are of sufficiently good quality.

³ For a 'lossy' air dielectric line, it is generally acceptable to assume that the shunt capacitance per unit length of line is the same as for a lossless line (i.e. $C = C_0$). Similarly, the shunt conductance can be assumed to be zero (i.e. $G = 0$).

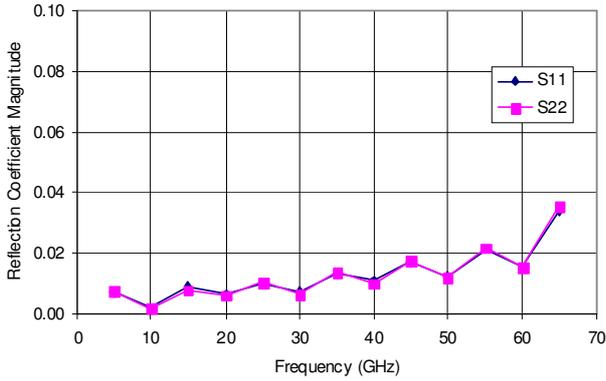


Figure 1: Measurements of the magnitude of the reflection coefficients of the 14.9 mm line

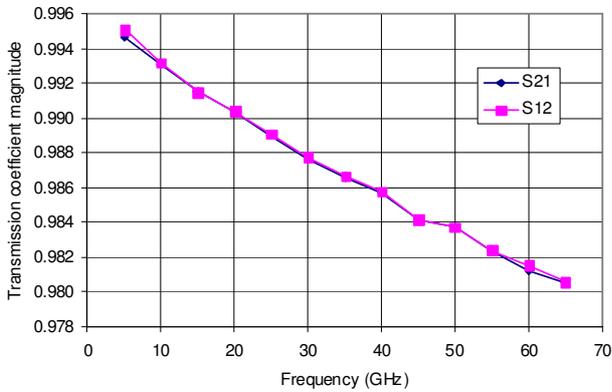


Figure 2: Measurements of the magnitude of the transmission coefficients of the 14.9 mm line

12. Use the values of β and Θ_{VTC} to provide an electrically determined length for the line, l_e , using

$$l_e = \frac{\Theta_{VTC}}{\beta} \quad (14)$$

13. Compare l_e with l_m to see if they are equivalent (to within the stated uncertainties).

III. RESULTS

The above method was tested using two beadless air lines, in the 1.85 mm line size, of nominal length 14.9 mm and 30.0 mm. The overall characteristic impedance of both these lines, derived from dimensional measurements of the radii of the lines' conductors, was found to be (49.3 ± 1.2) ohms. This

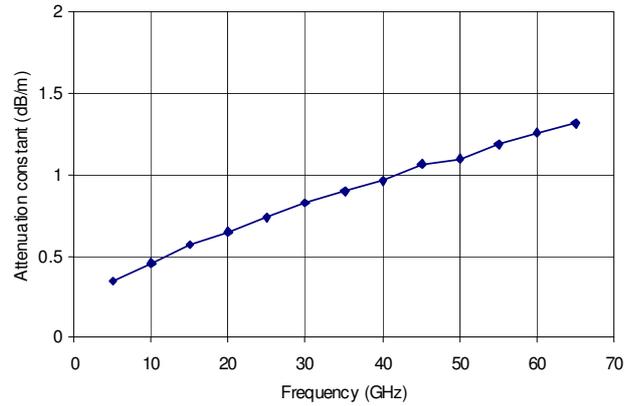


Figure 3: Determination of the attenuation constant of the 14.9 mm line

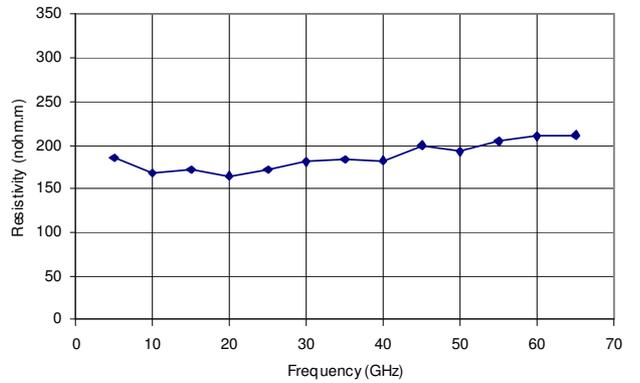


Figure 4: Determinations of the resistivity of the 14.9 mm line

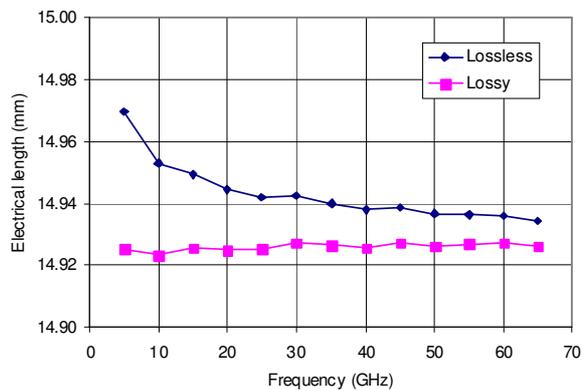


Figure 5: Electrical determinations of the length of the 14.9 mm line, comparing values where the loss in the line is included with values where the line is assumed lossless

average value of characteristic impedance, was considered to be sufficiently close to the system impedance (i.e. 50 ohms) for these lines to be considered well matched. The measured mechanical

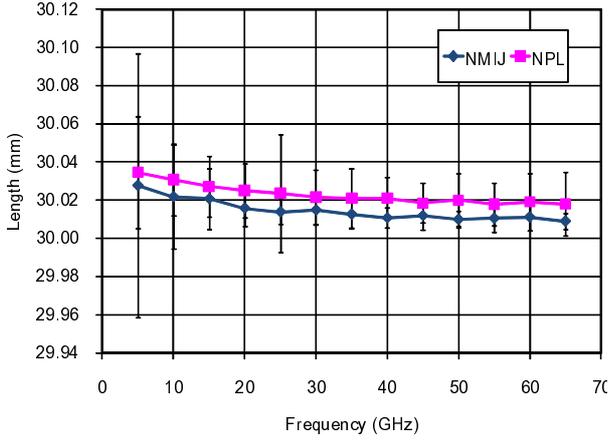


Figure 6: Electrical determination of length, made independently by NMIJ and NPL, of a line with nominal length 30 mm

length, l_m , of the lines was found to be (14.9136 ± 0.0019) mm and (29.9925 ± 0.0019) mm.

The measured S -parameters of the 14.9 mm line, as a function of frequency, are shown in Figures 1 and 2. Figure 1 shows the measured reflection coefficients, $|S_{11}|$ and $|S_{22}|$, and Figure 2 shows the associated transmission coefficients, $|S_{21}|$ and $|S_{12}|$. The results in Figure 1 show that the line is relatively well-matched at all measured frequencies. The measured attenuation constant of the line, derived from the transmission coefficient values, is shown in Figure 3. The associated determinations of resistivity for the line are shown in Figure 4.

The electrical determinations of the length of the line, l_e , are shown in Figure 5. This Figure also shows the equivalent electrically determined length if the line is assumed lossless. This shows clearly why it is important to take into account loss effects when using air lines as standards for phase.

IV. ESTABLISHING EQUIVALENCE

When comparing the electrically determined length with the mechanically determined length it is necessary to know the uncertainty in both determinations. This is so that the significance in the difference between the two determinations of length

can be assessed. This is done using the Normalized Error Ratio, Δ , where:

$$\Delta = \frac{|l_e - l_m|}{\sqrt{U^2(l_e) + U^2(l_m)}} \quad (15)$$

where $U(l_e)$ and $U(l_m)$ are the expanded uncertainties (at a 95% level of confidence) in l_e and l_m , respectively. In general, if $\Delta < 1$, then the difference between l_e and l_m is considered insignificant. If $\Delta > 1$, then the difference between l_e and l_m is considered significant.

V. UNCERTAINTY OF MEASUREMENT

The evaluation of the uncertainty in the determination of l_e can be established using the measurement model using a function, g :

$$Y = g(X) \quad (16)$$

In this case, the input quantities, X , are:

1. Center conductor radius, a
2. Outer conductor radius, b
3. Mechanical length, l_m
4. Transmission coefficient magnitude, $|VTC|$
5. Relative permittivity, ϵ_r
6. Frequency, f
7. Transmission coefficient phase, Θ_{VTC}

and so equation (16) will have the form:

$$l_e = g(a, b, l_m, |VTC|, \epsilon_r, f, \Theta_{VTC}) \quad (17)$$

A sensitivity analysis is applied to this model (following [12]) to establish an uncertainty budget for the determination of l_e . This budget shows the relative size of each of the uncertainty contributions and their overall combined effect on the determination of l_e .

The uncertainty is calculated using equation (18):

$$u(l_e) = \sqrt{\left(\frac{\partial l_e}{\partial a} \cdot u(a)\right)^2 + \left(\frac{\partial l_e}{\partial b} \cdot u(b)\right)^2 + \left(\frac{\partial l_e}{\partial l_m} \cdot u(l_m)\right)^2 + \left(\frac{\partial l_e}{\partial f} \cdot u(f)\right)^2 + \left(\frac{\partial l_e}{\partial \epsilon_r} \cdot u(\epsilon_r)\right)^2 + \left(\frac{\partial l_e}{\partial |S_{ij}|} \cdot u(|S_{ij}|) + \frac{\partial l_e}{\partial \Theta_{ij}} \cdot u(\Theta_{ij})\right)^2} \quad (18)$$

Table 1: Uncertainty budget for electrical length determination at 5 GHz

| Component of uncertainty | Estimated uncertainty, $u(x_i)$ | Sensitivity coefficient, $ \partial l_e / \partial x_i $ | Uncertainty in electrical length, $u(y_i)$ (μm) | |
|---|---------------------------------|--|--|--|
| Center conductor radius (μm) | 0.38 | 0.252 | 0.096 | |
| Outer conductor radius (μm) | 0.55 | 0.110 | 0.061 | |
| Mechanical length (μm) | 0.93 | 0.003 | 0.003 | |
| Relative permittivity | 0.000 038 | 14 990 | 0.572 | |
| Frequency (kHz) [0.05 ppm] | 0.25 | 0.006 | 0.002 | |
| VTC magnitude | 0.001 2 | 9 640 | 11.568 | |
| VTC phase (degrees) | 0.07 | 170 | 11.900 | |
| VNA measurements | | | 23.468 | |
| <hr/> | | | | |
| Combined standard uncertainty (μm) | | | 23.5 | |
| Expanded uncertainty, $k = 2$ (μm) | | | 47.0 | |

Table 2: Uncertainty budget for electrical length determination at 65 GHz

| Component of uncertainty | Estimated uncertainty, $u(x_i)$ | Sensitivity coefficient, $ \partial l_e / \partial x_i $ | Uncertainty in electrical length, $u(y_i)$ (μm) | |
|---|---------------------------------|--|--|--|
| Center conductor radius (μm) | 0.38 | 0.098 | 0.037 | |
| Outer conductor radius (μm) | 0.55 | 0.046 | 0.026 | |
| Mechanical length (μm) | 0.93 | 0.001 | 0.001 | |
| Relative permittivity | 0.000 038 | 14 990 | 0.572 | |
| Frequency (kHz) [0.05 ppm] | 3.25 | 0.0004 6 | 0.002 | |
| VTC magnitude | 0.008 8 | 730 | 6.424 | |
| VTC phase (degrees) | 0.53 | 13 | 6.890 | |
| VNA measurements | | | 13.314 | |
| <hr/> | | | | |
| Combined standard uncertainty (μm) | | | 13.3 | |
| Expanded uncertainty, $k = 2$ (μm) | | | 26.7 | |

Table 3: Electrical length determinations made by NMIJ and NPL for the 30 mm air line

| Frequency (GHz) | NMIJ | | NPL | | Δ |
|-----------------|------------|---------------|------------|---------------|----------|
| | l_e (mm) | $U(l_e)$ (mm) | l_e (mm) | $U(l_e)$ (mm) | |
| 5 | 30.027 6 | 0.097 9 | 30.034 6 | 0.041 5 | 0.07 |
| 65 | 30.008 9 | 0.006 0 | 30.018 0 | 0.023 6 | 0.37 |

Table 4: Electrical length determinations made by NMIJ and NPL for the 14.9 mm air line

| Frequency (GHz) | NMIJ | | NPL | | Δ |
|-----------------|------------|---------------|------------|---------------|----------|
| | l_e (mm) | $U(l_e)$ (mm) | l_e (mm) | $U(l_e)$ (mm) | |
| 5 | 14.920 6 | 0.097 9 | 14.915 8 | 0.045 6 | 0.04 |
| 65 | 14.920 5 | 0.005 5 | 14.923 6 | 0.018 1 | 0.16 |

It is recognised that the measurements of the magnitude and phase of the transmission coefficients are very likely to be correlated (since they are determined simultaneously, and by the same measurement process). Hence, the terms due to uncertainty in the transmission coefficient magnitude and phase are summed in equation (18) before being

combined with the other uncertainty contributions. (Linear summation provides a ‘safe’ (i.e. worst case) estimate of uncertainty when combining contributions that are expected to be correlated.)

Typical uncertainty budgets are shown in Tables 1 and 2, which relate to measurements made at 5 GHz and 65 GHz, respectively. The expanded uncertainty

in these tables is derived from the standard uncertainty using a coverage factor, k , of two. These tables show that the dominant sources of uncertainty are due to the magnitude and phase of the measured voltage transmission coefficients of the line. In fact, these Tables show that the other contributions listed in the uncertainty budgets can be ignored without underestimating the uncertainty in the determination of the electrical length of the line.

VI. REPRODUCIBILITY

To demonstrate the reproducibility of the method described in this paper, determinations of electrical length were made independently by two national measurement institutes (NMIs): NMIJ, Japan, and NPL, UK. Two beadless air lines were measured: one with nominal length 30 mm and the other with nominal length 14.9 mm.

Figure 6 shows the electrical length determinations obtained by both NMIs, as a function of frequency, for the line of nominal length 30 mm. Tables 3 and 4 summarize the results for both lines at 5 GHz and 65 GHz.

The level of agreement between these values can be assessed by performing a Normalized Error Ratio, Δ , calculation (of the form shown in equation (15)) for each set of values. Since all calculated values of Δ in Tables 3 and 4 show $\Delta < 1$, this demonstrates the reproducibility of the method for electrically determining the length of the line based on independent sets of measurements.

VII. SUMMARY

This paper has described a method for verifying measurements of the phase of transmission coefficients, based on determining the electrical length of precision beadless air lines. The method takes into account effects due to loss in the line thus resulting in a determination that is essentially independent of frequency (to within stated uncertainties). The electrical determination of length is compared subsequently with a mechanical determination of length (made using a dimensional measurement system with traceability to SI, i.e. the metre). If the determined electrical length agrees with the mechanical length (within stated uncertainties) then this verifies the transmission coefficient phase

measurements used to determine the electrical length of the line. In the case of a broadband frequency-domain instrument, such as a VNA, this procedure can be applied across the complete frequency range of the instrument, thus offering a full verification of the transmission phase measuring capabilities of the instrument.

There already exist techniques for verifying the magnitude of measurements made using VNAs (see, for example [13]). It is therefore proposed that the method described in this paper can be used in conjunction with methods given in [13] to provide a verification of both the magnitude and phase measurement capabilities of VNAs and related measuring instruments.

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