A generalised approach to the propagation of uncertainty in complex *S*-parameter measurements

Nick M Ridler and Martin J Salter
National Physical Laboratory, Teddington, Middlesex, UK

Abstract

This paper presents a generalised method for evaluating the effects of the uncertainty in complex S-parameter measurements on other, related, measurement quantities. The method utilises random numbers to simulate distributions for the measured S-parameters. These distributions are then passed through the measurement model to establish distributions in the output quantities (i.e. the measurands). Uncertainty estimates for the measurands are then obtained from the output distributions. This method finds particular application where the measurement model exhibits significant non-linearity and/or is too complicated to allow more conventional approaches to be applied. Examples of two such instances are given in the paper.

1 Introduction

Two recent ARFTG papers [1, 2] outlined an approach to calculating and propagating the uncertainty of measurement for complex-valued quantities (i.e. *S*-parameters). The approach extended the current internationally accepted guidelines used for scalar measurement quantities [3, 4]. However, the approach used to propagate the uncertainties in *S*-parameters (i.e. the input quantities) to other measurement quantities (i.e. the output quantities) described in [2] assumes that:

i)   the functional relationship[1] between input and output quantities is approximately linear;
ii) the functional relationship is sufficiently straightforward to allow all partial derivatives of the model to be determined. This is a requirement of the Law of Propagation of Uncertainty (LPU) used in [3, 4].

There are many instances in practical RF and microwave measurements where the above two conditions do not hold. For example, many measurement models can exhibit significant non-linearity. Indeed, even the simple example described in [2] involving a mismatch factor[2], *M*, applied during a power meter calibration, being of the form

$$M = 1 - \left| \Gamma \right|^2 \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(1)$$

is inherently non-linear. (In this expression, Γ is the complex-valued voltage reflection coefficient (VRC) of the power meter being calibrated.)

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[1] This functional relationship is described by the measurement model.

[2] This mismatch factor arises when the standard power meter and the power meter being calibrated are connected in turn to a stable generator and it is assumed that the generator and the standard power meter are reflectionless.

When LPU[3] is applied to this model the uncertainty in M, $u(M)$, becomes

$$u(M) = 2\sqrt{u^2(x)x^2 + u^2(y)y^2 + 2r(x,y)u(x)u(y)xy} \qquad \ldots\ldots\ldots\ldots(2)$$

where x and y are the real and imaginary components of Γ, $u(x)$ and $u(y)$ are the standard uncertainties in x and y and $r(x, y)$ is the correlation coefficient between x and y. A striking feature with this equation is that if the match of the unknown power meter is perfect (i.e. $x = y = 0$) then $u(M) = 0$. This is obviously unrealistic since no measured quantity can be determined with zero uncertainty (i.e. exactly) and so a more realistic method of propagating uncertainty in such a situation is needed.

In addition, the microwave literature is full of examples of complicated measurement models where the application of LPU becomes impractical because of the difficulty of determining the many partial derivatives associated with such complicated measurement models.

This paper presents an alternative, generalised, approach to the propagation of uncertainty in S-parameter measurements that does not require the above two conditions to hold. The approach is based on Monte Carlo Simulation (MCS) and is described in section 2 of this paper.[4] Section 3 shows that the method provides a more realistic determination of the uncertainty, $u(M)$, in the above power meter calibration example (i.e. $u(M) > 0$).

Section 4 shows how the method can be applied to a more complicated situation involving the measurement of a two-resistor power splitter such as is used to improve the effective output match of a microwave source [6, 7]. The three-port power splitter is measured on a two-port Vector Network Analyser (VNA) and the method of matrix renormalisation [8, 9] is used to determine the S-parameters of the splitter. Certain quantities of interest (input VSWR, effective output VSWRs and output tracking) are then computed from the S-parameters. In this situation, applying LPU would be difficult because of the complexity of evaluating the partial derivatives. MCS avoids these difficulties and also provides additional information about the distributions of the output quantities that is useful in determining the uncertainties for the quantities of interest.

2 Propagating uncertainty using MCS

In many practical situations, quantities of interest (such as various kinds of mismatch factors, effective output VSWRs of power splitters, etc) are computed from measured S-parameters. In such situations, there may be several input quantities and several output quantities.[5] In addition, the output quantities may

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[3] Throughout this paper, the term LPU is used to describe the use of only a first-order Taylor series approximation to the model. This is because it is felt that this is the approach most often used in practice. LPU involving higher-order approximations could also be used, but this is beyond the scope of this paper.

[4] This approach is in line with methods being proposed in a forthcoming document on uncertainty [5] intended to supplement the other international guidelines.

[5] It is useful here to introduce some terminology:
      i)   the measured quantities (in our case, the S-parameters) are called **input quantities;**
      ii)   the quantities of interest are called **output quantities;**
      iii)   the transformation relating the input quantities to the output quantities is called the **measurement model**.
This terminology applies more generally and is not restricted to measurements involving S-parameters.

be real-valued or complex-valued. (The input quantities, being *S*-parameters, are, by definition, complex-valued.)

In situations such as these, one is often interested in the uncertainty associated with the output quantities (e.g. for the purpose of conducting a conformance test of an output quantity against a specification). Thus it becomes necessary to be able to propagate the uncertainties associated with the input quantities (i.e. the *S*-parameters) through the measurement model to the output quantities.

This can be achieved using MCS involving the following five steps:

1. Specify the measurement model that relates the input quantities to the output quantities. In the above power meter calibration example, the model is

$$M = 1 - x^2 - y^2.$$

2. Assign a joint distribution to the input quantities of the measurement model. In the power meter calibration example, the input quantities are *x* and *y* (the real and imaginary components of Γ) and so a bivariate normal distribution is assigned to these variables with mean vector determined by the values of the input quantities and covariance matrix determined by the corresponding uncertainty matrix.

3. Generate a large random sample from the joint distribution of the input quantities (the 'input sample'). In the present example, each point in the input sample consists of a pair of values (x_i, y_i) representing the real and imaginary components of Γ.

4. Apply the measurement model to each point of the input sample to obtain a large random sample from the distribution of the output quantities (the 'output sample'). In the present example there is only the single real-valued output quantity, *M*.

5. Extract the required uncertainty information from the output sample. In our example, we are interested in the standard uncertainty in the output quantity *M* (obtained as the standard deviation of the output sample) and also in a coverage interval for *M* corresponding to a coverage probability of 95% (obtained from percentiles of the output sample). [6]

The use of MCS has a number of advantages over the conventional LPU approach. These include:

- MCS works with the full measurement model rather than a linear approximation to it;
- MCS avoids the need to evaluate the partial derivatives of the measurement model (i.e. the sensitivity coefficients), which can become very involved for more complicated measurement models;
- MCS provides more detailed information on the output quantities. The output sample provides an approximation to the distribution function of the output quantities. Thus MCS propagates distributions whereas LPU merely propagates uncertainties.

Since MCS involves a large number of computations, it generally requires a computer for its implementation. In order to implement the above strategy, it is necessary to be able to sample from the assigned input distribution. Where there are multiple input quantities (which may contain mutual

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[6] If there is more than one output quantity, mutual uncertainties (covariances) and correlation coefficients can also be estimated from the output sample.

uncertainties) then it is necessary to sample from a multivariate distribution. The most common multivariate distribution is the multivariate normal distribution. A procedure for sampling from a multivariate normal distribution is given in [5]. This procedure is described in more detail in [10].

3 Example 1: mismatch factor arising in power meter calibration

As was mentioned in Section 1 of this paper, applying LPU to obtain the uncertainty in the mismatch factor, M, at the origin in the complex VRC plane (i.e. when the power meter being calibrated is reflectionless) gives $u(M) = 0$. We now apply MCS to the same situation in order to provide a more realistic estimate of $u(M)$ near the origin in the complex VRC plane. Further away from the origin, the uncertainties obtained by using both methods are shown to agree thereby validating the use of LPU away from the origin.

Table 1 presents some uncertainties in mismatch factor, M, calculated using MCS with a sample size of 100,000 points. In the Table it is assumed that the VRC, Γ, lies on the line $x = y$ and that the uncertainties in Γ are given by $u(x) = u(y) = 0.005$, with mutual uncertainties described by a correlation coefficient $r(x, y) = + 0.6$.

Table 1: uncertainties in mismatch factor, M, calculated using MCS.
This example assumes: $x = y$; $u(x) = u(y) = 0.005$; and $r(x, y) = + 0.6$.

Magnitude of VRC, $|\Gamma|$	Mismatch factor, $\overline{M}$	Standard uncertainty, $u(M)$	Upper uncertainty, U_+	Lower uncertainty, U_-	Asymmetry, $|U_+ - U_-|$
0.001	1.00000	0.00006	0.00000	0.00012	0.00012
0.002	1.00000	0.00006	0.00000	0.00013	0.00013
0.005	0.99998	0.00009	0.00002	0.00017	0.00015
0.010	0.99990	0.00014	0.00010	0.00028	0.00018
0.020	0.99960	0.00026	0.00039	0.00049	0.00010
0.050	0.99750	0.00064	0.00113	0.00128	0.00015
0.100	0.99000	0.00127	0.00238	0.00250	0.00012

The summary information given about the measurand in this table is produced as follows:

- The value of the mismatch factor ($\overline{M}$) is obtained by evaluating the measurement model at the values of the input quantities $(\overline{x}, \overline{y})$;[7]
- The standard uncertainty in M, $u(M)$, is obtained as the standard deviation of the output sample;
- In general, expanded uncertainty intervals can be asymmetric about the mean measured value. Therefore, the upper and lower uncertainty points, U_+ and U_-, are treated separately. These points define a coverage interval for M extending from ($\overline{M}$ - U_-) to ($\overline{M}$ + U_+) corresponding to a coverage probability of 95%. The upper and lower uncertainties are obtained from a pair of percentiles of the output sample bracketing 95% of the values, such as the 2.5 percentile and the 97.5 percentile.[8] Using these percentiles we obtain:

$$U_+ = M_{0.975} - \overline{M}$$
$$U_- = \overline{M} - M_{0.025}$$

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[7] The mean of the output sample provides an alternative estimate of the value of the output quantity.

[8] In order to determine the percentiles, the numbers in the output sample are ordered in terms of their size.

where $M_{0.025}$ and $M_{0.925}$ denote the percentile values. The upper and lower uncertainties given in the Table are actually obtained from the shortest 95% interval defined by a pair of percentiles. The result can be expressed as

$$\overline{M}\left\{\pm\begin{matrix}U_+\\U_-\end{matrix}\right\}.$$

Figure 1 illustrates the use of MCS to calculate the uncertainty in the mismatch factor in the particular case used to produce the information in Table 1, for $|\Gamma| = 0.1$. A scatter plot in the complex VRC plane is used to represent the input sample and the output sample is represented by a histogram (scaled so that the total area of the histogram is equal to unity).
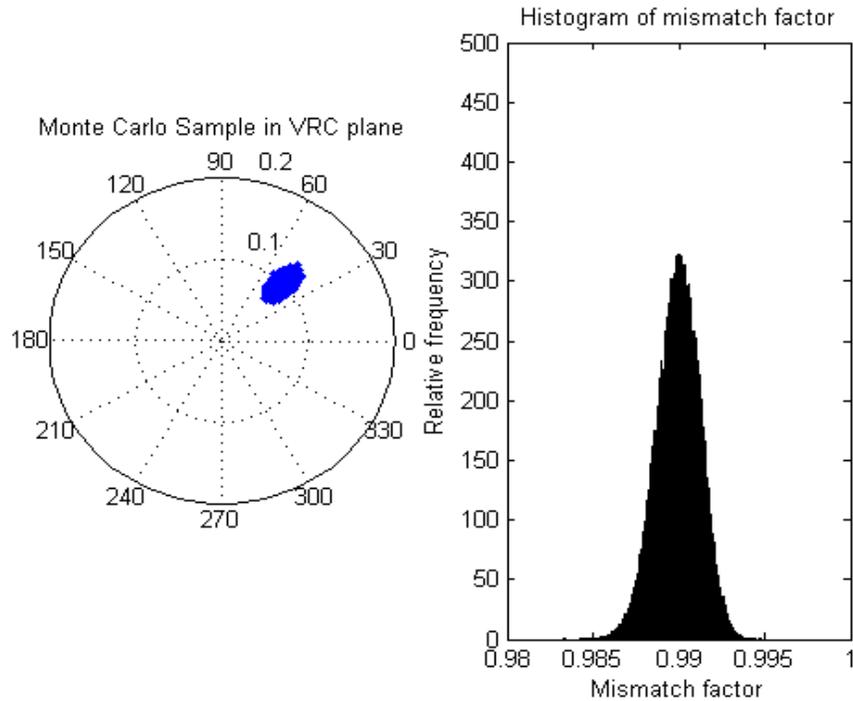

Figure 1: Scatter plot of MCS input sample (left) and corresponding (scaled) histogram for output sample (right) for M, with $|\Gamma| = 0.1$, $u(x) = u(y) = 0.005$ and $r(x, y) = + 0.6$.

Figure 2 illustrates standard uncertainties in mismatch factor obtained by LPU and MCS for the situation used to produce the information in Table 1, for $0 \leq |\Gamma| \leq 0.07$. This shows that the results obtained from the LPU and MCS uncertainty calculations diverge close to the origin. The LPU calculation predicts an uncertainty of zero at the origin whereas the MCS calculation predicts a more realistic (i.e. non-zero) uncertainty.[9] Further from the origin the two methods converge and both predict similar values of uncertainty.

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[9] The uncertainty predicted by MCS is still small compared with what might be expected based on practical experience. However, this is due to other components in the measurement model (i.e. the match of the standard power sensor and the signal generator) deliberately being ignored, in order to keep the example simple.
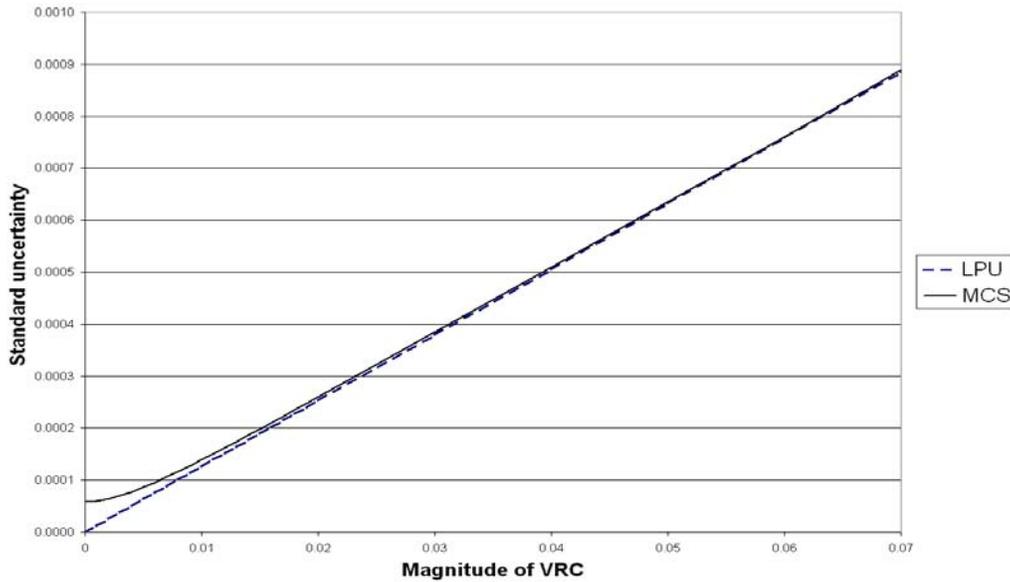
Figure 2: Standard uncertainty in mismatch factor M calculated using MCS and LPU
for VRC values on the line $x = y$, $u(x) = u(y) = 0.005$ and $r(x, y) = + 0.6$.

4 Example 2: measuring a three-port power splitter using a two-port VNA

Two-resistor power splitters (see Figure 3) are widely used to improve the effective output match of microwave sources through either a levelling loop or a ratio measurement [6, 7]. The parameters often quoted in the specification of such devices are the following measurands:

- Input VSWR at port 1;
- Effective output VSWR at port 2;
- Effective output VSWR at port 3;
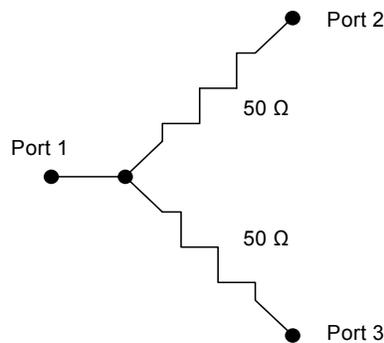- Output tracking between ports 2 and 3.

Figure 3: Schematic diagram of a two-resistor power splitter.

In order to check that a power splitter is operating within its specification, it is necessary to measure these quantities along with the associated uncertainties (usually at a level of confidence of 95%). One approach is to measure the S-parameters of the power splitter as a three-port device and to compute the specification measurands from the S-parameters.[10] If a two-port VNA is used, the splitter must be

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[10] An alternative approach to determining the effective output VSWRs has been given by Juroshek 11].

measured in a series of three two-port configurations (known as 'partial two-ports') with a terminating load attached to the unused port. These configurations are shown in Figure 4. The method of matrix renormalisation [8, 9] provides a means of determining the scattering matrix of the splitter; all that is required is that the VRC of the terminating load is known (e.g. from measurement).
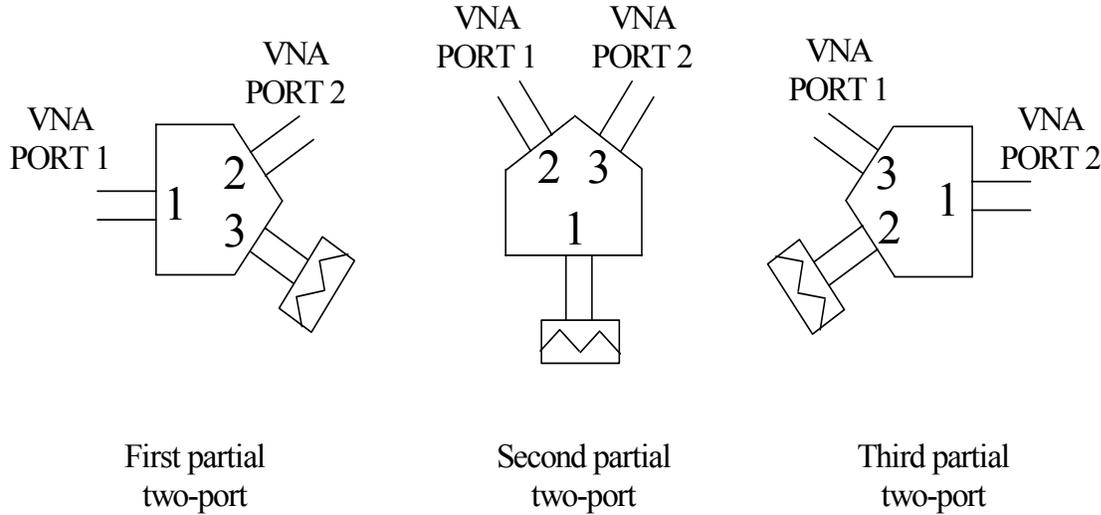

Figure 4: Typical sequence of connections to a three-port device, such as a power splitter, using a two-port VNA in order to determine all nine *S*-parameters.

4.1 Measurement model relating *S*-parameters to power splitter measurands

The input quantities to the measurement model are the (2×2) scattering matrices, $^{(1)}S$, $^{(2)}S$ and $^{(3)}S$, of the three partial two-ports (normalised to $Z_0 = 50$ ohms at both VNA test ports) and the VRC, ρ, of the terminating load (also normalised to $Z_0 = 50$ ohms). In general, the measurement of each partial two-port consists of measuring four complex *S*-parameters. In the case of a three-port device, three partial two-ports are measured making a total of 12 (i.e. 3×4) complex *S*-parameter measurements. In addition, the VRC of the terminating load is measured to facilitate impedance renormalisation to 50 ohms for the three-port's measurands. This makes a total of 13 (i.e. $12 + 1$) complex-valued input quantities. Since a complex-valued quantity is two dimensional, it follows that the vector of input quantities has dimension 26 (i.e. 2×13).

In practice, since a two-resistor power splitter is a reciprocal device (i.e. $S_{12} = S_{21}$), the four *S*-parameters for each partial two-port can be reduced to three measured *S*-parameters. This means that the 13 complex-valued input quantities are reduced to ten complex-valued input quantities so that the vector of input quantities has dimension 20 (i.e. 2×10).

For the purpose of this paper, the following simplified uncertainty structure will be assumed for the input quantities:

- The ten complex-valued input quantities are pairwise uncorrelated;
- The real and imaginary components of any given input quantity are uncorrelated;
- The standard uncertainty in the real and imaginary components of any given complex-valued input quantity is equal, although the standard uncertainty may vary from one input quantity to the next.

The output quantities of interest are the four real-valued measurands appearing in the power splitter specification. Therefore, the vector of output quantities is four-dimensional and the uncertainty is expressed as a (4×4) uncertainty matrix.[11]

The measurement model can be evaluated using the following steps:

- Compute the renormalised scattering matrices of the partial two-ports (normalised to Z_1 ohms at both ports, where Z_1 is the impedance of the terminating load) using the following matrix transformations [8, 9]:

$$
\begin{aligned}
{}^{(1)}S' &= {}^{(1)}S - \left(I_2 + {}^{(1)}S\right)\Gamma_2\left(I_2 - {}^{(1)}S\Gamma_2\right)^{-1}\left(I_2 - {}^{(1)}S\right) \\
{}^{(2)}S' &= {}^{(2)}S - \left(I_2 + {}^{(2)}S\right)\Gamma_2\left(I_2 - {}^{(2)}S\Gamma_2\right)^{-1}\left(I_2 - {}^{(2)}S\right) \qquad \text{.................(3)} \\
{}^{(3)}S' &= {}^{(3)}S - \left(I_2 + {}^{(3)}S\right)\Gamma_2\left(I_2 - {}^{(3)}S\Gamma_2\right)^{-1}\left(I_2 - {}^{(3)}S\right)
\end{aligned}
$$

where the (2×2) scattering matrices, ${}^{(1)}S'$, ${}^{(2)}S'$ and ${}^{(3)}S'$, are normalised to Z_1 at both ports, I_2 is the (2×2) identity matrix and:

$$
\Gamma_2 = \begin{bmatrix} \rho & 0 \\ 0 & \rho \end{bmatrix}.
$$

- Assemble the (3×3) scattering matrix of the splitter normalised to Z_1 at all three ports, S', from the elements of the three (2×2) scattering matrices, ${}^{(1)}S'$, ${}^{(2)}S'$ and ${}^{(3)}S'$, as follows:

$$
S' = \begin{bmatrix}
\dfrac{{}^{(1)}S'_{11} + {}^{(3)}S'_{22}}{2} & {}^{(1)}S'_{12} & {}^{(3)}S'_{21} \\[2ex]
{}^{(1)}S'_{21} & \dfrac{{}^{(1)}S'_{22} + {}^{(2)}S'_{11}}{2} & {}^{(2)}S'_{12} \\[2ex]
{}^{(3)}S'_{12} & {}^{(2)}S'_{21} & \dfrac{{}^{(2)}S'_{22} + {}^{(3)}S'_{11}}{2}
\end{bmatrix} \qquad \text{................(4)}
$$

- Renormalise the (3×3) scattering matrix of the splitter to $Z_0 = 50$ ohms at all three ports using the following matrix transformation [8, 9]:

$$
S = S' - \left(I_3 + S'\right)\Gamma_3\left(I_3 - S'\Gamma_3\right)^{-1}\left(I_3 - S'\right) \qquad \text{..................(5)}
$$

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[11] For some applications, the (3×3) scattering matrix of the splitter may be of more interest. This requires an (18×18) uncertainty matrix to fully express the uncertainty in these nine complex-valued measurands.

where S is the (3×3) scattering matrix normalised to Z_0 at all three ports, I_3 is the (3×3) identity matrix and:

$$\Gamma_3 = \begin{bmatrix} -\rho & 0 & 0 \\ 0 & -\rho & 0 \\ 0 & 0 & -\rho \end{bmatrix}$$

where

$$-\rho = \frac{Z_0 - Z_1}{Z_0 + Z_1}$$

is the VRC of Z_0 with respect to Z_1.

- Finally, calculate the power splitter measurands from the S-parameters of the splitter using the following equations. The input VSWR at port 1 is given in terms of S_{11} of the splitter:

$$(VSWR)_1 = \frac{1 + |S_{11}|}{1 - |S_{11}|} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(6)$$

Similarly, the effective output VSWRs at ports 2 and 3 are given by:

$$(VSWR)_2 = \frac{1 + |\Gamma_2|}{1 - |\Gamma_2|} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(7)$$

$$(VSWR)_3 = \frac{1 + |\Gamma_3|}{1 - |\Gamma_3|} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(8)$$

where the effective output VRCs, Γ_2 and Γ_3, are given in terms of the S-parameters of the splitter by:

$$\Gamma_2 = S_{22} - \frac{S_{12}S_{23}}{S_{13}}$$

and

$$\Gamma_3 = S_{33} - \frac{S_{13}S_{32}}{S_{12}} .$$

Finally, the output tracking, expressed in dB, is given by:

$$T = 20\log_{10}\left(\frac{|S_{21}|}{|S_{31}|}\right) \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(9)$$

4.2 Propagating uncertainties through the measurement model

The above measurement model (described by equations (3) to (9)) is relatively complicated. This makes LPU difficult to apply because of the need to evaluate all the partial derivatives of the measurement model. Under these circumstances the use of MCS becomes an attractive alternative. This avoids the need for the evaluation of partial derivatives and also potentially provides more reliable and extra information in the form of the distributions of the output quantities. MCS is applied to this problem by implementing the five steps outlined in section 2, as follows:

1. Equations (3) to (9) provide the measurement model;

2. A multivariate normal distribution is assigned to the 20-dimensional input vector with mean vector determined by the values of the input quantities and covariance matrix determined by the corresponding uncertainty matrix;

3. A large random sample (the input sample) is generated from the 20-dimensional joint distribution of the input quantities using the method given in [10];

4. The measurement model is applied to each point of the input sample to obtain a large random sample (the 'output sample') from the four-dimensional distribution of output quantities;

5. The required uncertainty information is extracted from the output sample. The values of the measurands are estimated either by evaluating the measurement model at the mean value of the input vector or by computing the mean vector of the output sample. The uncertainty matrix of the measurands is estimated by computing the uncertainty matrix of the output sample. The standard uncertainties are obtained by taking the positive square root of the diagonal elements of the uncertainty matrix. In order to obtain expanded uncertainties for the measurands (at a level of confidence of 95%), the four components of the output sample are separately ordered in terms of size. For each measurand, percentiles are calculated from the ordered data. The percentiles are used to establish the shortest interval (x_{lower}, x_{upper}) encompassing 95% of the distribution of the measurand.

   The expanded uncertainty in the measurand (at a level of confidence of 95%) can then be expressed as

$$\bar{x} \left\{ \pm \begin{matrix} x_+ \\ x_- \end{matrix} \right\}$$

   where $\bar{x}$ is the estimate of the measurand and

$$x_+ = x_{upper} - \bar{x}$$
$$x_- = \bar{x} - x_{lower}$$

4.3 Typical results

To demonstrate the above approach, a two-resistor power splitter, fitted with female Type-N connectors on each port, was measured in the frequency range 1 GHz to 18 GHz. The S-parameters of the partial two-ports and the VRC of the terminating load were measured using a two-port VNA.[12] The measurands were then determined and the uncertainty was propagated using MCS. As an example of the results obtained using this approach, Table 2 shows the effective output VSWR at port 3, at selected frequencies.

Table 2: Selected results for the effective output VSWR at port 3.

Frequency (GHz)	Value	Mean	Standard uncertainty	Upper uncertainty, U_+	Lower uncertainty, U_-	Asymmetry $|U_+ - U_-|$
3	1.0194	1.0194	0.0016	0.0031	0.0031	0.0000
6	1.0346	1.0347	0.0021	0.0041	0.0040	0.0001
9	1.0166	1.0168	0.0027	0.0054	0.0051	0.0003
12	1.0489	1.0491	0.0040	0.0081	0.0076	0.0005
15	1.0706	1.0709	0.0056	0.0114	0.0106	0.0008
18	1.0087	1.0134	0.0067	0.0174	0.0073	0.0101

The column labelled "Value" in this table gives estimates of the output quantity based on propagating the mean values of the input quantities (i.e. the measured S-parameters) through the measurement model. The column labelled "Mean" gives estimates of the output quantity based on the mean of the output sample produced by the MCS process. These two approaches to obtaining an estimate for an output quantity have been discussed elsewhere [3, 13]. In general, if data passes through a linear region of a measurement model, then the two approaches yield identical results. However, the two methods can produce different results when passing through a region of the measurement model that is significantly non-linear. For example, the above measurement model exhibits non-linear behaviour when the measured VSWR is close to one (i.e. the matched condition). This can be seen in the above results at 18 GHz where the VSWR is close to one (in the context of the size of the uncertainty) and there is subsequently a significant difference between the results quoted in the "Value" and "Mean" columns.

Histograms of the distributions of the four output quantities, obtained at 18 GHz, are shown in Figure 5 (normalised so that the total area under each histogram is equal to unity). An interesting feature in this Figure is that the histogram for the output VSWR at port 3 is visibly skewed (i.e. the tail on the right hand side of the distribution is significantly longer than on the left hand side). This is another indication that the measurement model is operating in a non-linear region. Under these circumstances estimates of the value of the measurand derived from the mean of the output sample should be treated with caution.

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[12] An LRL (Line-Reflect-Line) [12] technique was used to calibrate the VNA. This involved using a pair of non-insertable Line standards fitted with female connectors at both ends. (A single female short-circuit was used as the Reflect standard for both VNA test ports.) This produced two calibrated male test ports so that the S-parameters of the power splitter's partial two-ports could be measured directly by the VNA. The VRC of the male terminating load was measured by attaching a female-to-female adaptor to one of the VNA's male test ports. The S-parameters of the female-to-female adaptor were also determined by the VNA during the measurement process.
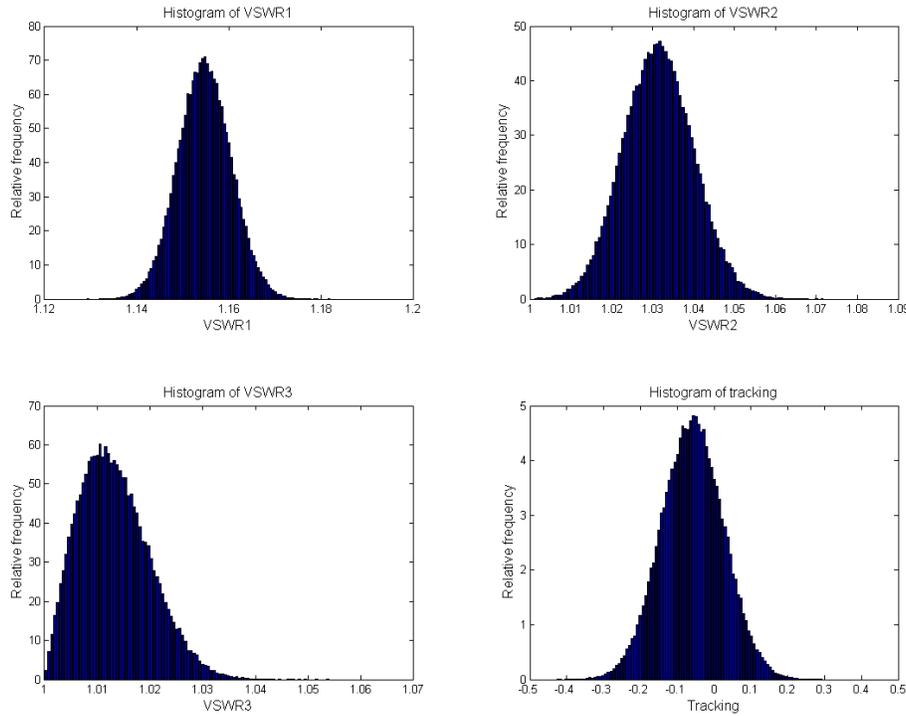
Figure 5: Normalised histograms of the components of
the MCS output sample comprising the four output quantities at 18 GHz.

The estimated correlation coefficients between the four measurands, obtained at 18 GHz, are given in Table 3. Values of correlation coefficient can range from −1 to +1, with zero indicating no correlation between a pair of measurands. It is, however, difficult to establish reliable and meaningful estimates of the 'true' value of a correlation coefficient. On this occasion, the values given in Table 3 indicate no strong correlation (either positive or negative) between these measurands, at this frequency, since all correlation coefficient values are closer to zero than they are to either +1 or −1.

Table 3: Correlation coefficients between the power splitter measurands at 18 GHz.

	VSWR1	VSWR2	VSWR3	Tracking
VSWR1	-	-0.1	+0.1	0.0
VSWR2		-	0.0	-0.3
VSWR3			-	-0.2
Tracking				-

5 Conclusions

This paper has presented a method for propagating uncertainty from complex *S*-parameter measurements to other measurement quantities. The method is considered to be a general approach to this problem in the sense that it can be applied to any measurement situation where a model for the measurement can be specified and suitable estimates for the input quantities can be established. The method then uses MCS to propagate the uncertainties in the input quantities through the measurement model to the output quantities, and hence achieve estimates for the measurands of interest.

The method has been applied successfully to a situation where it has been shown that the more traditional LPU method has failed to produce realistic estimates for the uncertainty in the measurand. This was the power meter calibration problem, discussed in example 1, where LPU gave an uncertainty of zero for the measurement of the mismatch of a perfectly matched power meter. An additional feature with this example was that the two methods showed good agreement away from the matched condition for the power meter, and so this provides a form of validation for the methods. The MCS method has then been applied successfully to a more complicated measurement situation (example 2), involving determining the specification parameters of a power splitter. Such a situation would be difficult to treat using LPU.

Finally, it is worth mentioning that the use of the MCS method, as implemented in the power splitter example, now forms part of a new measurement service offered by NPL for calibrating three-port devices, including power splitters, at RF and microwave frequencies.

6 Acknowledgement

7 References

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